UNIVERSITY OF SASKATCHEWAN
GE125.3 - Engineering Mechanics II
MIDTERM EXAMINATION

TIME: 2 HOURS **February 13, 2004**

INSTRUCTIONS:

1. Answer ALL questions. **All question carry equal marks**.
2. Only calculators, pens, pencils, and drawing aids are allowed for the exam.
3. Show your solution(s) in the space below the question. You may also write on the reverse side (if you need more space).
4. Make sure you supply your **Name**, **Student Number**, **Section Number**, and **Examination Room** in the space provided below. Also, place your name at the top of each sheet. ***You will be penalized for failing to do so***.

NAME: ____
First Name **Last Name**

STUDENT NUMBER: ____________________________

SECTION NUMBER: ____________________________

EXAMINATION ROOM: ____________________________

MARKS

1. ______________/25

2. ______________/25

3. ______________/25

4. ______________/25

TOTAL:____________/100

EXAMINATION ROOM LOCATIONS:
Section 02 (11:30 a.m. - 12:20 p.m.):
PHYSICS 107 (A - Kus)
PHYSICS 103 (L - Zz)

Section 04 (02:30 p.m. - 03:20 p.m.):
THORV 105 (A - Luz)
BIOL 106 (M - Zz)

INSTRUCTORS:
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Student Name: ___________________________________

1. The v-s graph for a go-cart traveling on a straight road is shown in **Fig. Q1**. Determine the acceleration of the go-cart at s = 50 m and s = 150 m. Draw a-s graph.

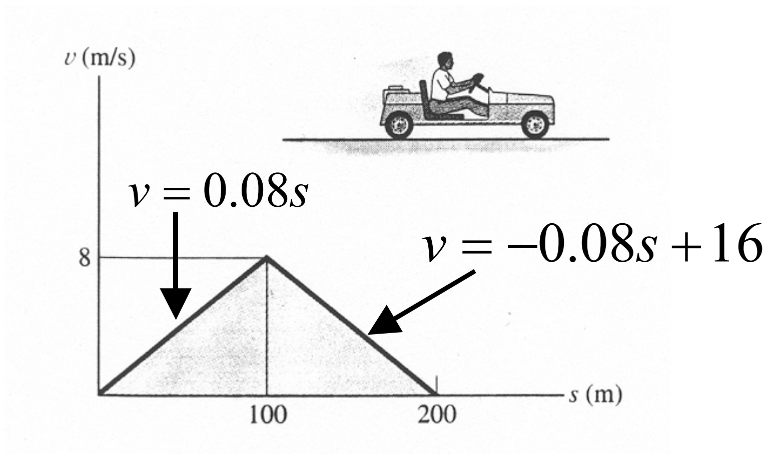

Fig. Q1

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2. The girl in **Fig. Q2** always throws the toys at an angle of 30º from point *A* as shown. Determine the time between throws so that toys strike the edges of the pool, *B* and *C*, at the same instant. With what speed must she throw each toy?

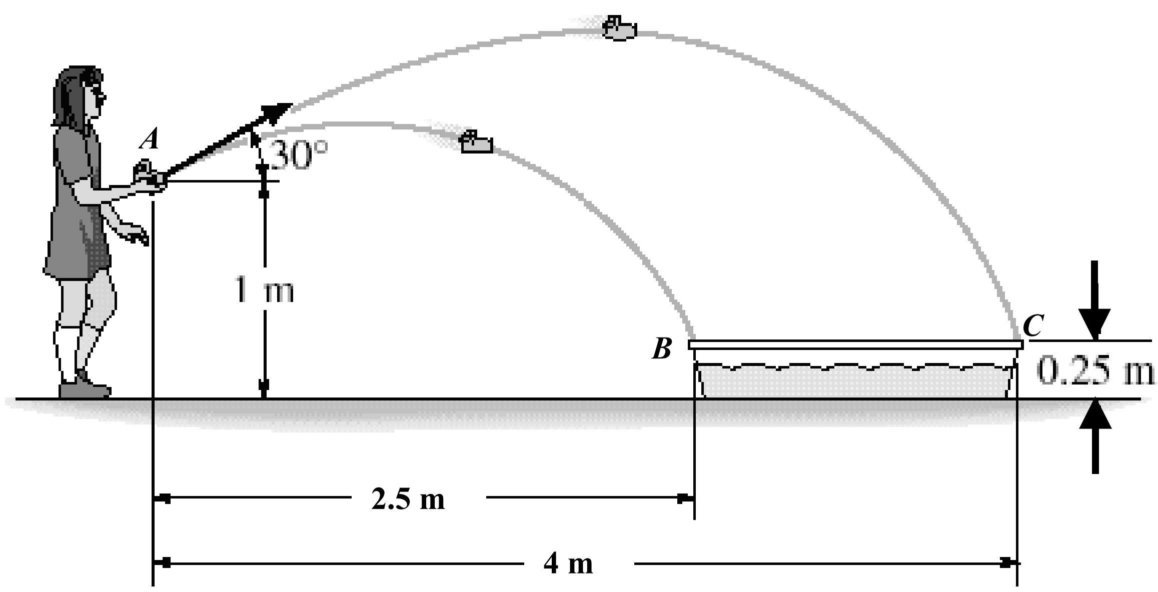

Fig. Q2

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3. The boy in **Fig. Q3** is riding in a car B. The car B turns such that its speed is $v_B = \left(0.5e^{2t}\right)$ m/s, where t is in seconds. If the car starts at $t = 0$ when $\theta = 0^\circ$, determine the forces acting on the boy when the arm AB rotates $\theta = 30^\circ$. The boy has a mass of 40 kg and the car B is moving in a horizontal plane. Neglect the size of the car.

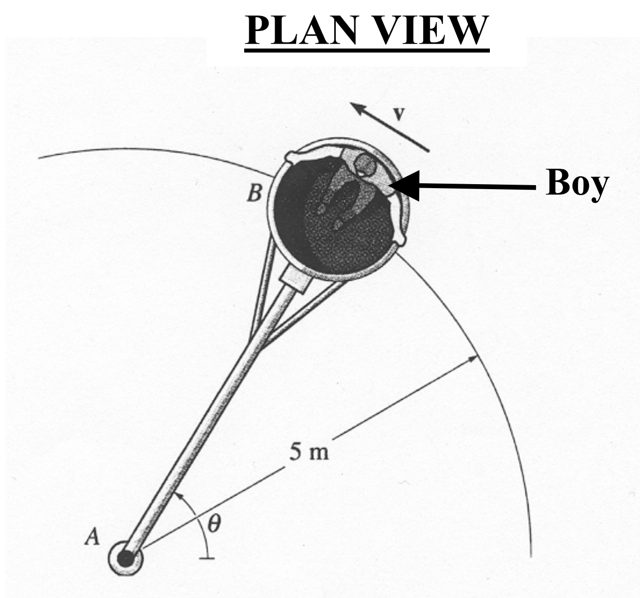

Fig. Q3

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4. In the assembly shown in **Fig. Q4**, a 5 kg mass ***A*** is connected to a 10 kg collar ***B*** by means of a rope and two small frictionless pulleys of negligible mass. It can be assumed that the collar is very small in size and slides without friction along the horizontal bar. If the system is released from rest, determine the initial tension in the rope and the initial acceleration of collar ***B***.

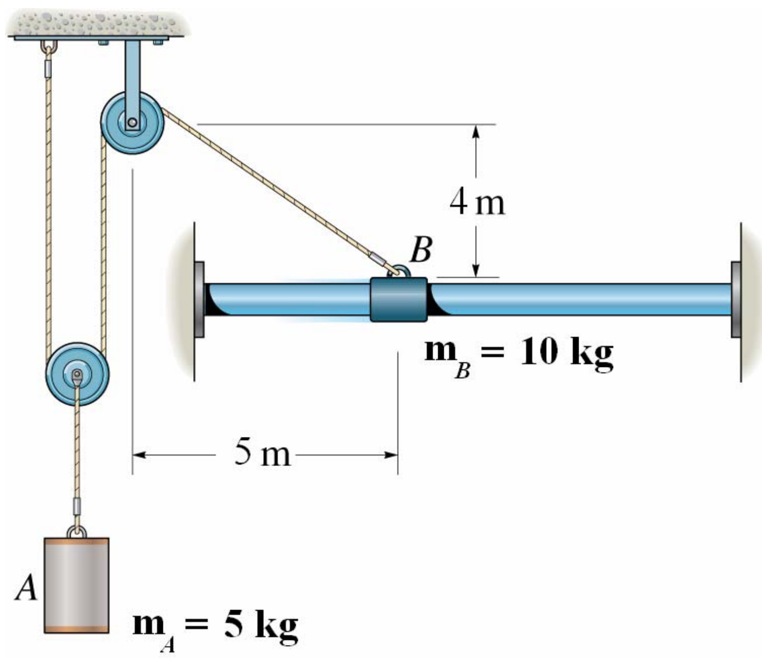

Fig. Q4.

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Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$ $\quad a_t = \dot{v} = v\dfrac{dv}{ds}$

$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $\quad I = \displaystyle\int r^2\,dm$

Parallel-Axis Theorem $\quad I = I_G + md^2$

Radius of Gyration $\quad k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1\text{-}2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \displaystyle\int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\displaystyle\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\displaystyle\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\displaystyle\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\displaystyle\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\displaystyle\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

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MATHEMATICAL EXPRESSIONS

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hyperbolic Functions

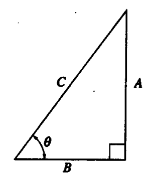
$\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\tanh x = \dfrac{\sinh x}{\cosh x}$

Trigonometric Identities

$\sin\theta = \dfrac{A}{C}$, $\csc\theta = \dfrac{C}{A}$

$\cos\theta = \dfrac{B}{C}$, $\sec\theta = \dfrac{C}{B}$

$\tan\theta = \dfrac{A}{B}$, $\cot\theta = \dfrac{B}{A}$



$\sin^2\theta + \cos^2\theta = 1$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\theta = \pm\sqrt{\dfrac{1 + \cos 2\theta}{2}}$, $\sin\theta = \pm\sqrt{\dfrac{1 - \cos 2\theta}{2}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Power-Series Expansions

$\sin x = x - \dfrac{x^3}{3!} + \cdots$ $\quad$ $\sinh x = x + \dfrac{x^3}{3!} + \cdots$

$\cos x = 1 - \dfrac{x^2}{2!} + \cdots$ $\quad$ $\cosh x = 1 + \dfrac{x^2}{2!} + \cdots$

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Derivatives

$\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}(\cot u) = -\csc^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec u) = \tan u \sec u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\csc u) = -\csc u \cot u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sin u) = \cos u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\sin u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan u) = \sec^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sinh u) = \cosh u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cosh u) = \sinh u\dfrac{du}{dx}$

INTEGRALS

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C, n \neq -1$

$\int \dfrac{dx}{a + bx} = \dfrac{1}{b}\ln(a + bx) + C$

$\int \dfrac{dx}{a + bx^2} = \dfrac{1}{2\sqrt{-ba}}\ln\left[\dfrac{a + x\sqrt{-ab}}{a - x\sqrt{-ab}}\right] + C, \quad ab < 0$

$\int \dfrac{x\,dx}{a + bx^2} = \dfrac{1}{2b}\ln(bx^2 + a) + C,$

$\int \dfrac{x^2\,dx}{a + bx^2} = \dfrac{x}{b} - \dfrac{a}{b\sqrt{ab}}\tan^{-1}\dfrac{x\sqrt{ab}}{a} + C, \quad ab > 0$

$\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\ln\left[\dfrac{a + x}{a - x}\right] + C, \quad a^2 > x^2$

$\int \sqrt{a + bx}\,dx = \dfrac{2}{3b}\sqrt{(a + bx)^3} + C$

$\int x\sqrt{a + bx}\,dx = \dfrac{-2(2a - 3bx)\sqrt{(a + bx)^3}}{15b^2} + C$

$\int x^2\sqrt{a + bx}\,dx = \dfrac{2(8a^2 - 12abx + 15b^2x^2)\sqrt{(a + bx)^3}}{105b^3} + C$

$\int \sqrt{a^2 - x^2}\,dx = \dfrac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2\sin^{-1}\dfrac{x}{a}\right] + C, \quad a > 0$

$\int x\sqrt{a^2 - x^2}\,dx = -\dfrac{1}{3}\sqrt{(a^2 - x^2)^3} + C$

$\int x^2\sqrt{a^2 - x^2}\,dx = -\dfrac{x}{4}\sqrt{(a^2 - x^2)^3} + \dfrac{a^2}{8}\left(x\sqrt{a^2 - x^2} + a^2\sin^{-1}\dfrac{x}{a}\right) + C, \quad a > 0$

$\int \sqrt{x^2 \pm a^2}\,dx = \dfrac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$

$\int x\sqrt{x^2 \pm a^2}\,dx = \dfrac{1}{3}\sqrt{(x^2 \pm a^2)^3} + C$

$\int x^2\sqrt{x^2 \pm a^2}\,dx = \dfrac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \dfrac{a^2}{8}x\sqrt{x^2 \pm a^2} - \dfrac{a^4}{8}\ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$

$\int \dfrac{dx}{\sqrt{a + bx}} = \dfrac{2\sqrt{a + bx}}{b} + C$

$\int \dfrac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$

$\int \dfrac{dx}{\sqrt{a + bx + cx^2}} = \dfrac{1}{\sqrt{c}}\ln\left[\sqrt{a + bx + cx^2} + x\sqrt{c} + \dfrac{b}{2\sqrt{c}}\right] + C, \quad c > 0$

$= \dfrac{1}{\sqrt{-c}}\sin^{-1}\left(\dfrac{-2cx - b}{\sqrt{b^2 - 4ac}}\right) + C, \quad c > 0$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int x\cos(ax)\,dx = \dfrac{1}{a^2}\cos(ax) + \dfrac{x}{a}\sin(ax) + C$

$\int x^2\cos(ax)\,dx = \dfrac{2x}{a^2}\cos(ax) + \dfrac{a^2x^2 - 2}{a^3}\sin(ax) + C$

$\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + C$

$\int xe^{ax}\,dx = \dfrac{e^{ax}}{a^2}(ax - 1) + C$

$\int \sinh x\,dx = \cosh x + C$

$\int \cosh x\,dx = \sinh x + C$